VISIT…

Appendix D: Dielectric Behavior

Electrical insulators are also known as dielectrics. Most ionic solids and molecular solids are insulators because of the negligible concentration of conduction electrons or holes.

Consider two metal (conductor) plates connected to the two ends of a battery of voltage V (Figure D.1). Assuming the electrical resistance of the connecting wires is negligible, the potential between the two metal plates is V. Assume that the medium between the plates is a vacuum.

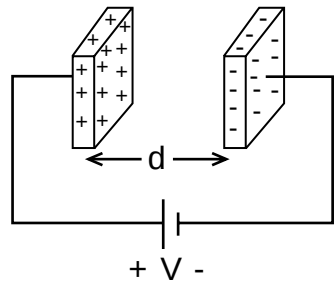

FIGURE D.1 A pair of positively charged and negatively charged conductor plates in vacuum.

There are a lot of conduction electrons in the metal plates and metal wires. The positive end of the battery attracts conduction electrons, making the left plate positively charged (charge = +Q). The negative end of the battery repels conduction electrons, making the right plate negatively charged (charge = –Q). Let the area of each plate be A. the magnitude of charge per unit area of each plate is known as the "charge density" (D_o).

$$D_o = \frac{Q}{A} \tag{D.1}$$

The electric field E between the plates is given by

$$E = \frac{V}{d} \tag{D.2}$$

©2001 CRC Press LLC

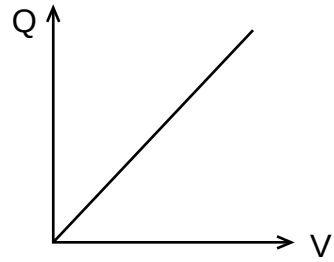

FIGURE D.2 Plot of charge Q vs. potential V. The slope = $C_o = \frac{\varepsilon_o A}{d}$.

where d is the separation of the plates. When E = 0, $D_o = 0$. In fact, D is proportional to E. Let the proportionality constant be ε_o. Then

$$D_o = \varepsilon_o E \tag{D.3}$$

ε_o is called the "permittivity of free space." It is a universal constant.

$$\varepsilon_o = 8.85 \times 10^{-12}\ \text{C/(V.m)}$$

Eq. (D.3) is known as Gauss's law.

Just as D_o is proportional to E, Q is proportional to V. The plot of Q versus V is a straight line through the origin (Figure D.2), with

$$\text{Slope} = C_o = \frac{Q}{V} = \frac{\varepsilon_o EA}{Ed} = \frac{\varepsilon_o A}{d} \tag{D.4}$$

C_o is known as the "capacitance." Its unit is Coulomb/Volt, or Farad (F). In fact, this is the principle behind a parallel-plate capacitor.

Next, consider that the medium between the two plates is not a vacuum, but an insulator whose center of positive charge and center of negative charge coincide when V = 0. When V > 0, the center of positive charge is shifted toward the negative plate, while the center of negative charge is shifted toward the positive plate. Such displacement of the centers of positive and negative charges is known as "polarization." In the case of a molecular solid with polarized molecules (e.g., HF), the polarization in the molecular solid is due to the preferred orientation of each molecule such that the positive end of the molecule is closer to the negative plate. In the case of an ionic solid, the polarization is due to the slight movement of the cations toward the negative plate, and that of the anions toward the positive plate. In the case of an atomic solid, the polarization is due to the skewing of the electron clouds toward the positive plate.

When polarization occurs, the center of positive charge sucks more electrons to the negative plate, causing the charge on the negative plate to be $-\kappa Q$, where $\kappa > 1$ (Figure D.3). Similarly, the center of negative charge repels more electrons away

©2001 CRC Press LLC

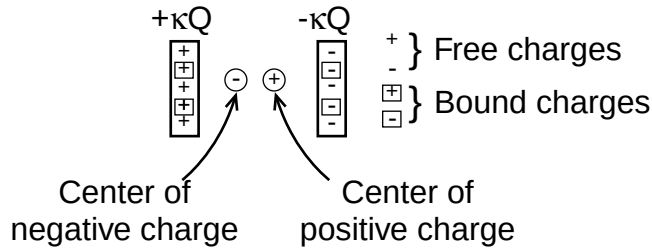

FIGURE D.3 A pair of positively charged and negatively charged conductor plates in a medium with relative dielectric constant κ.

from the positive plate, causing the charge on the positive plate to be κQ. κ is a unitless number called the "relative dielectric constant." Its value at 1 MHz (10^6 Hz) is 2.3 for polyethylene, 3.2 for polyvinyl chloride, 6.5 for Al_2O_3, 3000 for $BaTiO_3$, and 78.3 for water.

The charges in the plates when a vacuum is between the plates are called "free charges" (magnitude = Q on each plate). The extra charges in the plates when an insulator is between the plates are called the "bound charges" (magnitude = κQ – Q = (κ – 1)Q on each plate).

When an insulator is between the plates, the charge density is given by

$$D_m = \kappa D_o = \frac{\kappa Q}{A} \tag{D.5}$$

Using Eq. (D.3), Eq. (D.5) becomes

$$D_m = \kappa \varepsilon_o E = \varepsilon E \tag{D.6}$$

where $\varepsilon \equiv \kappa\varepsilon_o$; ε is known as the dielectric constant, whereas κ is known as the relative dielectric constant. Hence, ε and ε_o have the same unit.

When an insulator is between the plates, the capacitance is given by

$$C_m = \frac{\kappa Q}{V} = \frac{\kappa \varepsilon_o EA}{Ed} = \frac{\kappa \varepsilon_o A}{d} = \kappa C_o \tag{D.7}$$

From Eq. (D.7), the capacitance is inversely proportional to d, so capacitance measurement provides a way to detect changes in d (i.e., to sense strain).

Mathematically, the polarization is defined as the bound charge density, so that it is given by

$$\begin{aligned} P &= D_m - D_o \\ &= \kappa \varepsilon_o E - \varepsilon_o E \\ &= (\kappa - 1)\, \varepsilon_o E \end{aligned} \tag{D.8}$$

©2001 CRC Press LLC

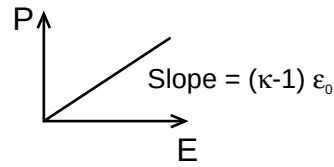

FIGURE D.4 Plot of polarization **P** vs. electric field **E**.

The plot of P versus E is a straight line through the origin, with a slope of $(\kappa - 1)\,\varepsilon_o$ (Figure D.4).

The ratio of the bound charge density to the free charge density is given by

$$\frac{\kappa Q - Q}{Q} = \kappa - 1 \tag{D.9}$$

The quantity $\kappa - 1$ is known as the "electric susceptibility" (χ). Hence,

$$\chi = \kappa - 1 = \frac{P}{\varepsilon_o E} \tag{D.10}$$

The dipole moment in the polarized insulator is given by

$$(\text{bound charge})d = (\kappa - 1)\,Qd \tag{D.11}$$

since the bound charges are induced by the dipole moment in the polarized insulator.

The dipole moment per unit volume of the polarized insulator is thus given by

$$\frac{\text{Dipole moment}}{\text{Volume}} = \frac{(\kappa - 1)Qd}{Ad} = \frac{(\kappa - 1)Q}{A} = P \tag{D.12}$$

Therefore, another meaning of polarization is the dipole moment per unit volume.

Now consider that the applied voltage V is an AC voltage, so that it alternates between positive and negative values. When $V > 0$, $E > 0$, and the polarization is one way. When $V < 0$, $E < 0$, and the polarization is in the opposite direction. If the frequency of V is beyond about 10^{10} Hz, the molecules in the insulator cannot reorient themselves fast enough to respond to V (i.e., dipole friction occurs), so κ decreases (Figure D.5). When the frequency is beyond 10^{15} Hz, even the electron clouds cannot change their skewing directions fast enough to respond to V, so κ decreases further. The minimum value of κ is 1, which is the value for vacuum.

In making capacitors, one prefers to use insulators with very large values of κ so that C_m is large. However, one should be aware that the value of κ depends on the frequency. It is challenging to make a capacitor that operates at very high frequencies.

©2001 CRC Press LLC

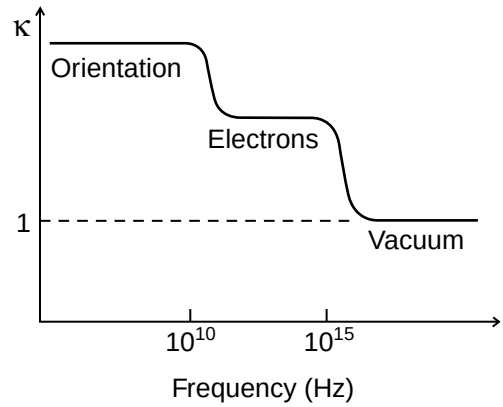

FIGURE D.5 Dependence of the relative dielectric constant κ on the frequency.

In AC condition, it is mathematically more convenient to express E and D_m in complex notation, i.e.,

$$E = \hat{E}e^{i\omega t} = \hat{E}(\cos \omega t + i \sin \omega t) \tag{D.13}$$

and

$$D_m = \hat{D}_m e^{i(\omega t - \delta)} = \hat{D}_m[\cos(\omega t - \delta) + i\sin(\omega t - \delta)] \tag{D.14}$$

where ω is the angular frequency (which is related to the frequency f by the equation $\omega = 2\pi f$; angular frequency has the unit radians/s, whereas frequency has the unit cycle/s, or Hertz (Hz)), δ is the dielectric loss angle (which describes the lag of D_m with respect to E), $\hat{E}$ is the amplitude of the E wave, and $\hat{D}_m$ is the amplitude of the D_m wave. If $\delta = 0$, E and D_m are in phase and there is no lag. Not only are E and D_m complex, ε is complex too. Eq. (D.6) then becomes

$$\hat{D}_m e^{i(\omega t - \delta)} = \varepsilon \hat{E} e^{i\omega t}$$

Hence,

$$\varepsilon = \frac{\hat{D}_m}{\hat{E}} e^{-i\delta} = \frac{\hat{D}_m}{\hat{E}}(\cos\delta - i\sin\delta) \tag{D.15}$$

The quantity $\hat{D}_m/\hat{E}$ is known as the "static dielectric constant." From Eq. (D.15), the loss factor (or loss tangent or dielectric loss or dissipation factor) tan δ is given by

$$\tan\delta = -\frac{\text{Imaginary part of } \varepsilon}{\text{Real part of } \varepsilon} \tag{D.16}$$

©2001 CRC Press LLC

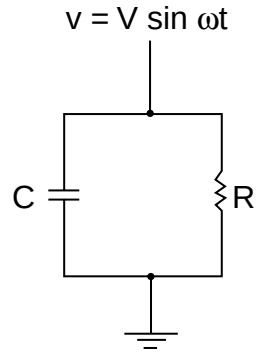

FIGURE D.6 A nonideal capacitor modeled as a capacitor C and a resistor R in parallel.

In another convention, the dissipation factor is defined as tan δ, while the dielectric loss factor is defined as κ tan δ.

By definition, $\varepsilon = \kappa\varepsilon_o$, so Eq. (D.16) can be written as

$$\tan\delta = -\frac{\text{Imaginary part of } \kappa}{\text{Real part of } \kappa} \tag{D.17}$$

The loss factor tan δ is more commonly used than δ itself to describe the extent of lag between D_m and E. This is because tan δ relates to the energy loss. The value of tan δ at 1 MHz (10^6 Hz) is 0.0001 for polyethylene, 0.05 for polyvinyl chloride, and 0.001 for Al_2O_3.

Lag occurs when the insulator is not a perfect insulator. A nonideal capacitor can be modeled as a capacitor C in parallel with a resistor (DC) R, as shown in Figure D.6. If the insulator is perfect, R will be infinite and thus disappear from Figure D.6. The current i_c through the capacitor is given by

$$i_c = \frac{dQ}{dt} = C\frac{dv}{dt} \tag{D.18}$$

where v is the AC voltage across both capacitor and resistor. Let

$$v = V \sin \omega t \tag{D.19}$$

where V is the amplitude of the voltage wave of angular frequency ω, which is related to the frequency f and period T by

$$\omega = 2\pi f = \frac{2\pi}{t} \tag{D.20}$$

Substituting Eq. (D.19) into Eq. (D.18),

©2001 CRC Press LLC

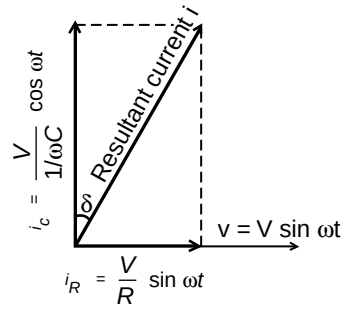

FIGURE D.7 Phasor diagram for the nonideal capacitor of Figure D.6.

$$i_c = C\frac{dv}{dt} = \omega CV\cos\omega t \qquad (D.21)$$
$$= \frac{V}{1/(\omega C)}\cos\omega t$$

Since

$$\sin\left(\omega t + \frac{\pi}{2}\right) = \sin\omega t\cos\frac{\pi}{2} + \cos\omega t\sin\frac{\pi}{2}$$
$$= \cos\omega t$$

Eq. (D.21) becomes

$$i_c = \frac{V}{1/(\omega C)}\sin\left(\omega t + \frac{\pi}{2}\right) \qquad (D.22)$$

Comparison of Eq. (D.22) with Eq. (D.19) shows that i_c leads v by a phase angle of π/2, or 90°. This pertains to the current through the capacitor only.

The current through the resistor R is

$$i_R = \frac{v}{R} = \frac{V}{R}\sin\omega t \qquad (D.23)$$

i_R and v are in phase.

The phase relationship is conventionally described by a phasor diagram (Figure D.7), which shows that i_R is in phase with v, i_C is 90° ahead of v, and the resultant current i is ahead of v by $\phi = 90° - \delta$. From Figure D.7,

$$\tan\delta = \frac{V/R}{V\omega C} = \frac{1}{\omega CR} \qquad (D.24)$$

©2001 CRC Press LLC

The electrical energy stored in the perfect capacitor is

$$\text{Energy stored} = \int_0^{\tau} v i_C dt \tag{D.25}$$

$$= \int_0^{\tau} V^2 \omega C \sin\omega t \cos\omega t dt$$

$$= \int_0^{\tau} \frac{V^2 \omega C}{2} \sin 2\omega t dt$$

$$= -\frac{V^2 \omega C}{4\omega(2)} [\cos 2\omega]_0^{\tau}$$

$$= -\frac{1}{4} CV^2 (\cos 2\omega t - 1)$$

The maximum value of this energy is $\frac{1}{2}CV^2$, and this occurs when cos 2ωt = –1. The higher is C, the greater the energy loss.

The loss of energy per cycle due to conduction through the resistor R is

$$\text{Energy loss} = \frac{V^2}{R} \int_0^{2\pi/\omega} \sin\omega t \sin\omega t dt \tag{D.26}$$

$$= \frac{V^2}{\omega R} \int_0^{2\pi} \frac{1}{2}(1 - \cos 2\omega t) d(\omega t)$$

$$= \frac{V^2}{\omega R} \left[\frac{1}{2}\left(\omega t - \frac{1}{2}\sin 2\omega t\right)\right]_0^{2\pi}$$

$$= \frac{V^2}{\omega R} \left[\frac{1}{2}(2\pi - 0 - 0 + 0)\right]$$

$$= \frac{V^2 \pi}{\omega R}$$

The smaller is R, the greater is the energy loss. When R = ∞, R becomes an open circuit and disappears from the circuit of Figure D.6.

$$\frac{\text{Energy lost per cycle}}{2\pi \times \text{maximum energy stored}} = \frac{V^2\pi/\omega R}{2\pi CV^2/2} \tag{D.27}$$

$$= \frac{1}{\omega CR} = \tan\delta$$

Hence, tan δ is related to the energy loss. Energy loss is undesirable for most piezoelectric applications.

Dipole friction causes energy loss such that the greatest loss occurs at frequencies at which dipole orientation can almost, but not quite, occur. This is shown in Figure D.8, which corresponds to Figure D.5. A peak in tan δ occurs at each step decrease in κ as the frequency increases.

©2001 CRC Press LLC

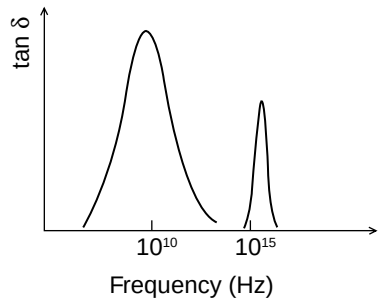

FIGURE D.8 Dependence of tan δ on the frequency.

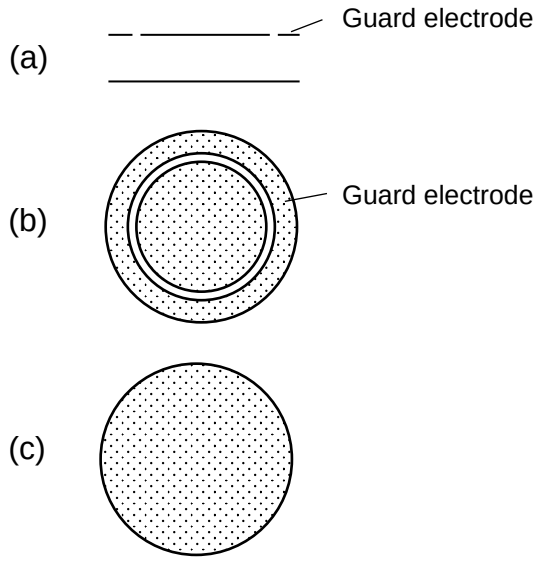

FIGURE D.9 Three-electrode method of dielectric measurement: (**a**) Side view, (**b**) top view, (**c**) bottom view.

Dielectric measurement refers to measurement of the relative dielectric constant κ of a dielectric material. Parallel plate geometry is used because the capacitance increases with plate area. This geometry involves sandwiching the sample between electrical conductor plates, applying a voltage across the parallel plates, and measuring the impedance across the plates. The measurement is best conducted using the three-electrode method, which involves one of the plates being one electrode, and the other plate consisting of an outer electrode and an inner electrode that are electrically unconnected (Figure D.9). The outer electrode, called the "guard electrode," ensures that the electric flux lines across the inner electrode and the other plate are all perpendicular to the plates so that a cross-section of the sample parallel to the plates is equipotential. On the other hand, the electric flux lines across the

©2001 CRC Press LLC

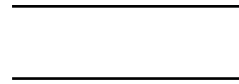

FIGURE D.10 Two-electrode method of dielectric measurement, side view.

outer electrode and the other plate partly curve outward. In contrast to the three-electrode method is the inferior two-electrode method (Figure D.10), which suffers from the outward curving of the electric flux lines near the edge of the sample.

Dielectric measurements can be performed as functions of frequency as well as voltage across the plates. The variations of the impedance with frequency and voltage provide information on the origin of dielectric behavior. The temperature may also be varied to yield even more information.

©2001 CRC Press LLC